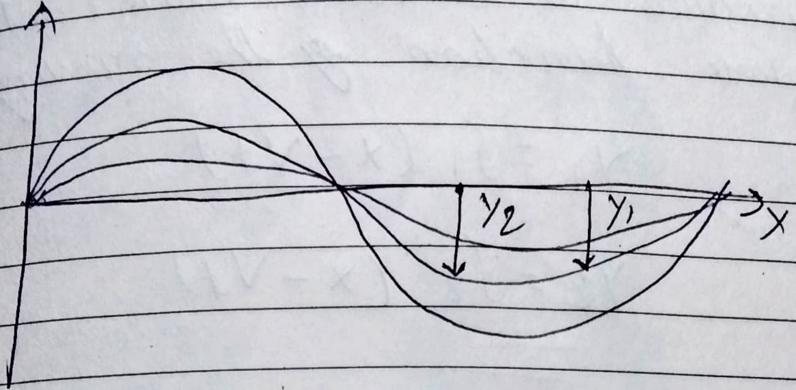
Superposition of waves →

According to the principle of superposition the resultant displacement of a number of waves in a medium at a particular point is the vector sum of the individual displacement produced by each of the waves at that point



Principle of superposition of waves

consider two waves travelling simultaneously along the same stretched string in opposite directions, as shown in the figure above. We can see images of waveforms in the string at each instant of time. It is observed that the net displacement of any element of the string at a given time is the algebraic sum of the displacements due to each wave.

Let us say two waves are travelling alone, and the displacement of any element of these two waves can be represented by $Y_1(X,t)$ and $Y_2(X,t)$.

When these two waves overlap, the resultant displacement can be given as $y(x, t)$

Mathematically,

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

As per the principle of superposition, we can add the overlapped waves algebraically to produce a resultant wave. Let us say the wave function of the moving waves are

$$y_1 = f_1(x - vt)$$

$$y_2 = f_2(x - vt)$$

$$\text{———}$$

$$y_n = f_n(x - vt)$$

Then, the wave function describing the disturbance in the medium can be described as

$$y = f_1(x - vt) + f_2(x - vt) + \text{-----} + f_n(x - vt)$$

or $y = \Sigma i = 1$ to $n = f_i(x - vt)$

Let us consider a wave travelling along a stretched given by

$$y_1(x, t) = A \sin(kx - \omega t)$$

and another wave, shifted

from the first by a phase ϕ

given as $y_2(x,t) = A \sin(Kx - \omega t + \phi)$

From the equations, we can see that both the waves have the same angular frequency, the same angular wave number K, and hence the same wavelength and the same amplitude A.
Now applying the superposition principle the resultant wave is the algebraic sum of the two constituent waves and has displacement

$$y(x,t) = A \sin(Kx - \omega t) + A \sin(Kx - \omega t + \phi)$$

The above equation can be written as

$$y(x,t) = 2A \cos(\phi/2) \cdot \sin(Kx - \omega t + \phi/2)$$

The resultant wave is a sinusoidal wave, travelling in the positive x direction, where the phase angle is half of the phase difference of the individual waves and the amplitude as $[2\cos\phi/2]$ times the amplitudes of the original waves.

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